UG-C-2284 **BMS-21X/ BMC-21X**

U.G. DEGREE EXAMINATION – DECEMBER, 2023.

Mathematics

Second Year

GROUPS AND RINGS

Time : 3 hours Maximum marks : 70

PART A — ($3 \times 3 = 9$ marks)

Answer any THREE questions out of Five questions in 100 words.

All questions carry equal marks.

1. Write a note on equivalence relation.

2. Show that in a group G, $x^2 = x$ iff $x = e$.

3. Define isomorphism of a group.

4. Write a short note on maximal ideal.

5. What is meant by euclidean domain?

PART B — ($3 \times 7 = 21$ marks)

Answer any THREE questions out of Five questions in 200 words.

Al! questions carry equal marks.

6. Let $f : A \to B$, $g : B \to C$ be bijection and then prove that $g \circ f : A \to C$ is also a bijection.

7. Show that a non-empty subset H of a group G is a subgroup of G, iff $a, b \in H \Rightarrow a\ b^{-1} \in H$.

8. Let H and k be two finite subgroups of a group G, then prove that $|H\ K| = \frac{|H|\ |K|}{|H \cap K|}$.

9. Show that the intersection of two subrings of a ring R is a subring of R.

10. Show that the field of complex number is not an ordered field.

PART C — ($4 \times 10 = 40$ marks)

Answer any FOUR questions out of Seven questions in 500 words.

All questions carry equal marks.

11. Show that any partition of a set S determines an equivalence relation ρ such that the members of the partition are precisely the equivalence classes define by ρ.

12. Show that the union of two subgroup of a group G is a subgroup, if one is contained in the other.

13. State and prove Lagrange's Theorem.

14. State and prove Fundamental theorem of homomorphism of a group.

15. Let R and R' be rings and $f : R \to R'$ be an isomorphism. Then prove that the following:

    (a) R is commutative $\Rightarrow R'$ is commutative

    (b) R is an ring with identity $\Rightarrow R'$ is a ring with identity

    (c) R is an integral domain $\Rightarrow R'$ is an integral domain

    (d) R is a field $\Rightarrow R'$ is a field.

16. State and prove Cayley's theorem.

17. Prove that any euclidean domain R is a unique factorization domain.

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UG–C–2286 **BMS–23X/ BMC–22X**

U.G. DEGREE EXAMINATION – DECEMBER, 2023.

Mathematics

Second Year

CLASSICAL ALGEBRA AND NUMERICAL METHODS

Time : 3 hours Maximum marks : 70

PART A — (3 × 3 = 9 marks)

Answer any THREE questions out of Five questions in 100 words.

All questions carry equal marks.

1. Prove that
$$\frac{a-x}{a}+\frac{1}{2}\left(\frac{a-x}{a}\right)^2+\frac{1}{3}\left(\frac{a-x}{a}\right)^3+\ldots=\log a-\log x\ .$$

2. If α and β are the roots of $2x^2+3x+5=0$, find $\alpha+\beta, \alpha\beta$.

3. What is the condition for the convergence of the iterative method for solving $x=\phi(x)$?

4. Prove that:

   (a) $E = 1 + \Delta$

   (b) $E = (1 - \nabla)^{-1}$

5. Given $u_0 = 1, u_1 = 15, u_2 = 57$ find $\frac{dy}{dx}$ at $x = 2$.

PART B — ($3 \times 7 = 21$ marks)

Answer any THREE questions out of Five questions in 200 words.

All questions carry equal marks.

6. Sum the series $1 - \frac{1}{4} + \frac{1.3}{4.8} - \frac{1.3.5}{4.8.12} + ...\infty$.

7. Solve the equation $x^3 - 12x^2 + 39x - 28 = 0$ whose roots are in A.P.

8. Find the value of $\sqrt{5}$ by Newton-Raphson method.

9. Use Lagrange's interpolation formula to fit a polynomial to the data and find the value of y when $x = 2$

x	0	1	3	4
y	–12	0	6	12

10. Solve $y' = x + y$ given $y(1) = 0$ and get $y(1.1), h = 0.1$ by Taylor's series method.

PART C — ($4 \times 10 = 40$ marks)

Answer any FOUR questions out of Seven questions in 500 words.

All questions carry equal marks.

11. Prove that $\sum_{n=0}^{\infty} \frac{5n+1}{(2n+1)!} = \frac{e}{2} + \frac{2}{e}$.

12. Solve $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$.

13. Solve the system of equations by Gauss Jordan method

$2x + 3y - z = 5$.

$4x + 4y - 3z = 3$

$2x - 3y + 2z = 2$

14. Using Laplace-Everett's formula find $\log 337.5$ given that

x	310	320	330	340	350	360
$\log x$	2.491	2.505	2.518	2.531	2.544	2.556

15. By dividing the range into six equal parts, evaluate $\int_0^6 \frac{1}{1+x} dx$ using Trapezoidal rule, Simpson's $\frac{1}{3}^{rd}$ rule and Simpson's $\frac{3}{8}^{th}$ rule.

16. (a) Find the value of x when $y = 7$ by Lagrange's Interpolation formula.

x	1	3	4
y	4	12	19

(b) Express $3x^3 - 2x^2 + 7x - 6$ in factorial polynomials and get their successive forward differences tanking $h = 1$.

17. If α, β, γ are the roots of the equation $x^3 - px^2 + qx - r = 0$ find the value of

(a) $\Sigma\alpha^2$ (b) $\Sigma\alpha^3$

(c) $\Sigma\alpha^2\beta$ (d) $\Sigma\alpha^2\beta^2$

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