UG-CS-1340 **BMSS–11/ BMSS–11C**

U.G. DEGREE EXAMINATION – FEBRUARY 2025.

Mathematics

First Year

ALGEBRA

Time : 3 hours Maximum marks : 70

SECTION A — (3 × 3 = 9 marks)

Answer any THREE questions.

1. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$. Find the value of $\sum \alpha^2 \beta$.

2. Diminish by 3 the roots of the equation $x^5 - 4x^4 + 3x^3 - 4x + 6 = 0$.

3. Define Hermitian matrix and give an example.

4. Define prime number and give an example.

5. State any two properties of Congruence Modulus.

SECTION B — (3 × 7 = 21 marks)

Answer any THREE questions.

6. Remove the fractional coefficients from the equation $2x^3 + \frac{3}{2}x^2 - \frac{1}{8}x - \frac{3}{16} = 0$.

7. Solve $x^3 - 6x - 9 = 0$ by Cardan's method.

8. If A is a square matrix of order n, then prove that $A + \overline{A^T}$ is Hermitian and $A - \overline{A^T}$ is skew Hermitian.

9. Find the number and sum of all the divisors of 360.

10. Find a number having the remainders 5, 4, 3, 2 when divided by 6, 5, 4, 3 respectively.

SECTION C — (4 × 10 = 40 marks)

Answer any FOUR questions.

11. Solve $4x^4 - 20x^3 + 33x^2 - 20x + 4 = 0$.

12. Sum the series to infinity $\frac{15}{16} + \frac{15 \cdot 12}{16 \cdot 24} + \frac{15 \cdot 21 \cdot 27}{16 \cdot 24 \cdot 32} + \cdots$.

13. Find the eigen values and eigen vectors of $\begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & -2 \\ 1 & -1 & 2 \end{pmatrix}$.

14. Prove that the product of r consecutive integers is divisible by $r!$

15. State and Prove Fermat's theorem.

16. Solve $6x^6 - 25x^5 + 31x^4 + 31x^2 + 25x - 6 = 0$.

17. Find the positive root of the equation $x^3 - 2x^2 - 3x - 4 = 0$ correct to three places of decimals using Homer's method.

UG-CS-1341 **BMSSE-11/ BMSSE-11C**

U.G. DEGREE EXAMINATION – FEBRUARY 2025.

Mathematics

First Year

TRIGONOMETRY

Time : 3 hours Maximum marks : 70

SECTION A — (3 × 3 = 9 marks)

Answer any THREE questions.

1. If $\cos\theta + i\sin\theta = x$, then find the value of $\cos 5\theta$.

2. Find the expansion of $\sin 5\theta$ in terms of $\sin\theta$.

3. Find the real and imaginary parts of $\sin(x + iy)$.

4. Find the general value of logarithm of $(x + iy)$.

5. Write the sum of the series upto n terms $\sin\alpha + \sin 2\alpha + \sin 3\alpha + \ldots$

SECTION B — (3 × 7 = 21 marks)

Answer any THREE questions.

6. Prove that

$$\cos^6\theta = \frac{1}{32}\left[\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10\right].$$

7. If $\frac{\sin\theta}{\theta} = \frac{5045}{5046}$, show that θ is equal to $1^\circ 58'$ nearly.

8. If $\sin(A + iB) = x + iy$, then prove that

(a) $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$ and

(b) $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$

9. Prove that $i^i = e^{-(4n+1)\frac{\pi}{2}}$, where n is any integer.

10. Sum the series $\sin^3\frac{\theta}{3} + 3\sin^3\frac{\theta}{3^2} + 3^2\sin^3\frac{\theta}{3^3} + \ldots$

SECTION C — (4 × 10 = 40 marks)

Answer any FOUR questions.

11. Prove that

$$\sin^4\theta\cos^3\theta = \frac{1}{64}\left[\cos 7\theta - \cos 5\theta - 3\cos 3\theta + 3\cos\theta\right]$$

12. Prove that

$$\frac{\cos 9\theta}{\cos\theta} = 256\cos^8\theta - 576\cos^6\theta + 432\cos^4\theta - 120\cos^2\theta + 9$$

13. If $\cos(x+iy) = r(\cos\alpha + i\sin\alpha)$, prove that $y = \frac{1}{2}\log\left[\frac{\sin(x-\alpha)}{\sin(x+\alpha)}\right]$.

14. Prove that

$$Log\,(1+\cos 2\theta + i\sin 2\theta) = \log(2\cos\theta) + i(2n\pi + \theta)$$

15. Sum to infinity the series $\cos\theta\cos\theta + \cos^2\theta\cos 2\theta + \cos^3\theta\cos 3\theta + \ldots$

16. Evaluate $\lim_{\theta \to 0} \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta - 1}$.

17. Find the sum to infinity the series, Where c is positive and $c < 1$.

$$c\sin\alpha - \frac{1}{2}c^2\sin 2\alpha + \frac{1}{3}c^3\sin 3\alpha + \ldots$$

UG-CS-1343 **BMSS–21/ BMSS-21C**

U.G. DEGREE EXAMINATION – FEBRUARY 2025.

Mathematics

First Year

DIFFERENTIAL CALCULUS

Time : 3 hours Maximum marks : 70

SECTION A — (3 × 3 = 9 marks)

Answer any THREE questions.

1. Write n^{th} derivative of $\frac{1}{(ax+b)^2}$.

2. If $x^3 + y^3 = 3axy$, find $\frac{dy}{dx}$.

3. Define envelope.

4. Write the formula to find the angle of intersection of two curves in polar coordinates.

5. Write the different types of asymptotes.

SECTION B — (3 × 7 = 21 marks)

Answer any THREE questions.

6. Find n^{th} derivative of $\frac{2x+1}{(2x-1)(2x+3)}$.

7. Transform $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}$ into polar coordinates.

8. Find the envelope of the family of lines $x\cos\alpha + y\sin\alpha = p$.

9. Find the slope of the tangent with the initial line for the cardioid $r = a(1 - \cos\theta)$ at $\theta = \frac{\pi}{6}$.

10. Write the working rule for finding asymptotes to a curve.

SECTION C — (4 × 10 = 40 marks)

Answer any FOUR questions.

11. If $y^{\frac{1}{m}} + y^{\frac{-1}{m}} = 2x$, show that $(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$.

12. Find the extreme values of $xy(a - x - y)$.

13. Find the envelope of the family of curves $\frac{x^3}{a^3 \cos\theta} + \frac{y^3}{b^3 \sin\theta} = 1$, where θ is a parameter.

14. Find $\frac{ds}{d\theta}$ and $\frac{ds}{dr}$ for the cardioid $r = a(1 + \cos\theta)$.

15. Find the asymptotes of

$$x^4y + 2x^3y^2 - x^2y^3 - 2xy^4 - x^3y + xy^3 + x^2 + y^2 + 1 = 0$$

16. Find the shortest distance from the point (1, 0, –2) to the plane $x + 2y + z = 4$.

17. Find the radius of curvature at the point $\left(\frac{a}{4}, \frac{a}{4}\right)$ to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$.

UG-CS-1344 **BMSSE-21/ BMSSE-21C**

U.G. DEGREE EXAMINATION – FEBRUARY 2025.

Mathematics

Second Year

ANALYTICAL GEOMETRY

Time : 3 hours Maximum marks : 70

SECTION A — (3 × 3 = 9 marks)

Answer any THREE questions.

1. Write the condition for the line $y = mx + c$ to be a tangent to the parabola $y^2 = 4ax$.

2. Define conjugate diameters.

3. Write the equation of plane in normal form..

4. Find the equation of straight line joining the points $(1, -1, 2)$ and $(4, 2, 3)$.

5. Find the centre and radius of the sphere $16x^2 + 16y^2 + 16z^2 - 16x - 8y - 16z - 55 = 0$.

SECTION B — (3 × 7 = 21 marks)

Answer any THREE questions.

6. Find the equation of the chord joining the points $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ on the parabola $y^2 = 4ax$.

7. Find the condition that the pair of lines $Ax^2 + 2Hxy + By^2 = 0$ to be conjugate diameters of the hyperbola.

8. Find the equation of the plane which passes through the points $(1, 0, -1)$ and $(2, 1, 1)$ and parallel to the line joining the points $(-2, 1, 3)$ and $(5, 2, 0)$.

9. Prove that the points $(3, 2, 4)$, $(4, 5, 2)$ and $(5, 8, 0)$ are collinear.

10. Find the equation of the sphere tangential to the plane $x - 2y - 2z = 7$ at $(3, -1, -1)$ and passing through the point $(1, 1, -3)$.

SECTION C — (4 × 10 = 40 marks)

Answer any FOUR questions.

11. Find the equation of the asymptotes of the hyperbola $3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0$.

12. Prove that the parallelogram formed by the tangents at the extremities of conjugate diameters of hyperbola has its vertices lying on the asymptotes and is of constant area.

13. Find the equations of the planes through the points $(0, 4, -3)$ and $(6, -4, 3)$ which cuts off from the axes intercepts whose sum is zero.

14. Find the shortest distance between the lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-15}{-5}$.

15. Find the equation of the sphere passing through the circle $x^2 + y^2 + z^2 + 2x + y - 3z - 1 = 0$, $-x + 2y + 5z + 7 = 0$ and cutting orthogonally the sphere $x^2 + y^2 + z^2 - 7x - 3y + 5z - 6 = 0$.

16. Show that the locus of the point of intersection of perpendicular tangents to a conic is a circle or a straight line.

17. Find the image or reflection of the point (5, 3, 2) in the plane $x + y - z = 5$.

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UG-CS-1346 **BMSS–31**

U.G. DEGREE EXAMINATION – FEBRUARY 2025.

Mathematics

Third Semester

INTEGRAL CALCULUS

Time : 3 hours Maximum marks : 70

SECTION A — (3 × 3 = 9 marks)

Answer any THREE questions.

1. Obtain a reduction formula for $\int x^n e^{ax}\,dx$, where n is a positive integer.

2. Evaluate $\int_0^a \int_0^{\sqrt{ay}} xy\,dx\,dy$.

3. Evaluate $\int_0^\infty \frac{1}{a^2 + x^2}\,dx$.

4. Define gradient of a scalar point function.

5. State Stoke's theorem.

SECTION B — (3 × 7 = 21 marks)

Answer any THREE questions.

6. Evaluate $\int x^4 (\log x)^3 \, dx$.

7. By changing into polar coordinates evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx \, dy$.

8. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

9. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, –1, 2).

10. If $\vec{F} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$. Evaluate $\int_C \vec{F}.d\vec{r}$. from (0, 0, 0) to (1, 1, 1) along the path $x = t, \, y = t^2, \, z = t^3$.

SECTION C — (4 × 10 = 40 marks)

Answer any FOUR questions.

11. Obtain a reduction formula for $\int \sin^n x dx$ when n it is a positive integer.

12. Find the area bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

13. Obtain the relationship Beta and Gamma functions.

14. Show that $\vec{F} = (2x + yz)\vec{i} + (4y + zx)\vec{j} - (6z - xy)\vec{k}$ is irrotational and hence the scalar potential of $\vec{F}$.

15. Verify Gauss's theorem for $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ taken over the region bounded by the planes $x = 0, x = a, y = 0, y = a, z = 0, z = a$.

16. Prove that $\nabla.(r^n \vec{r}) = (n + 3)r^n$.

17. Evaluate $\iint_s \vec{F}.\hat{n}ds$ where $\vec{F} = z\vec{i} + x\vec{j} - y^2 z\vec{k}$ and S is the surface of the cylinder $x^2 + y^2 = 1$ included in the first octant between the planes and $z = 0$ and $z = 2$.

UG-CS-1347 **BMSS–32**

U.G. DEGREE EXAMINATION – FEBRUARY 2025.

Mathematics

Third Semester

DIFFERENTIAL EQUATIONS

Time : Three hours Maximum marks : 70

SECTION A — (3 × 3 = 9 marks)

Answer any THREE questions in 100 words.

All questions carry equal marks.

1. Solve $xp^2 - yp + p^2 + 1 = 0$.

2. Solve $(D^2 + 5D + 6)y = 0$.

3. Find the complementary function of $(2x+5)^2 \frac{d^2y}{dx^2} - (2x+5)\frac{dy}{dx} + 8y = 0$.

4. Show that the differential equation $x\frac{d^2y}{dx^2}+2x\frac{dy}{dx}+2y=0$ is exact.

5. Form the partial differential equation by eliminating a and b from $z=(x^2+a)(y^2+b)$.

SECTION B — (3 × 7 = 21 marks)

Answer any THREE questions in 200 words.

All questions carry equal marks.

6. Solve $\frac{dy}{dx}=\frac{x-y}{x+y}$.

7. Solve $(D^2+16)y=\cos 4x$.

8. Find the solution of $\frac{d^2y}{dx^2}+n^2y=\sec nx$, using the method of variation of parameter.

9. Show that the differential equation $(x^3-4)\frac{d^3y}{dx^3}+9x^2\frac{d^2y}{dx^2}+18x\frac{dy}{dx}+6y=6$ is exact and find its first integral.

10. Solve the partial differential equation $p+q=x+y$.

SECTION C — (4 × 10 = 40 marks)

Answer any FOUR questions in 500 words.
All questions carry equal marks.

11. Solve $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$

12. Solve $(D^2 + 1)y = x\cos x$.

13. Solve $(1+x^2)^3 \frac{d^2y}{dx^2} + 2x(1+x^2)^2 \frac{dy}{dx} + (1+x^2)y = 3x$ by means of the substitution $x = \tan\theta$.

14. Solve $\frac{dx}{(x^2 - yz)} = \frac{dy}{(y^2 - zx)} = \frac{dz}{z^2 - xy}$.

15. Using Charpit's method, evaluate $pxy + pq + qy = yz.\ 92$.

16. Solve $(D^2 - 2D + 5)y = e^{2x}\sin x$.

17. Solve partial differential the equation $p^2 + q^2 = z^2(x^2 + y^2)$.

UG-CS-1349 **BMSS–41**

U.G. DEGREE EXAMINATION – FEBRUARY 2025.

Mathematics

Fourth Semester

TRANSFORM TECHNIQUES

Time : 3 hours Maximum marks : 70

PART A — (3 × 3 = 9 marks)

Answer any THREE questions out of Eight questions.

All questions carry equal marks.

1. Find $L[\sinh 4t]$.

2. Evaluate $L^{-1}\left[\frac{s+4}{s^2+9}\right]$.

3. Find the Euler constant a_0 for the function $f(x) = x(2\pi - x)$ in $(0, 2\pi)$.

4. Find the Fourier sine transform of $\frac{1}{x}$.

5. State the convolution theorem for the Fourier transform.

PART B — ($3 \times 7 = 21$ marks)

Answer any THREE questions out of Eight questions.

All questions carry equal marks.

6. Find the Laplace transform of $f(t) = \begin{cases} \sin t & when\ 0 < t < \pi \\ 0 & when\ \pi < t < \pi \end{cases}$.

7. Find $L^{-1}\left[\frac{s}{(s+3)^2}\right]$.

8. Expand the function $f(x) = \sin x,\ 0 < x < \pi$ in Fourier cosine series.

9. Using Fourier integral representation show that $\int_0^\infty \frac{\omega \sin x\omega}{1+\omega^2} = \frac{\pi}{2} e^{-x}$.

10. Solve $\frac{\partial y}{\partial x} = 2\frac{\partial y}{\partial t} + Y,\ Y(x, 0) = 6e^{-3x}$ which is bounded for $x > 0, t > 0$.

PART C — ($4 \times 10 = 40$ marks)

Answer any FOUR questions out of Five questions in 500 words.

All questions carry equal marks.

11. (a) Evaluate the integral using Laplace transform $\int_0^\infty \frac{(\cos 6t - \cos 4t)}{t} dt$.

(b) Find $L[t \cos 3t]$.

12. Find :

(a) $L^{-1}\left[\log\left(\frac{s+3}{s-4}\right)\right]$

(b) $L^{-1}\left[\frac{s-3}{s^2+4s+13}\right]$.

13. Find the Fourier series for $f(x) = x^2$ in $-\pi \le x \le \pi$ and deduce that

(a) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots = \frac{\pi^2}{6}$

(b) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$.

14. Show that the Fourier transform of $e^{-\frac{x^2}{2}}$ is $e^{-\frac{s^2}{2}}$.

15. Using Laplace transform, solve $\frac{d^2y}{dt^2}+2\frac{dy}{dt}-3y=\sin t$, given that $y=0$, $\frac{dy}{dt}=0$ when $t=0$.

16. State and prove the convolution theorem for Laplace transform.

17. Find the Fourier transform of $f(x)$ given by $f(x)=\begin{cases}1 & for\ |x|<2\\ 0 & for\ |x|>2\end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin x}{x}dx$ and $\int_0^\infty \left(\frac{\sin x}{x}\right)^2 dx$.

UG-CS-1350 **BMSS–42**

U.G. DEGREE EXAMINATION – FEBRUARY 2025.

Mathematics

Fourth Semester

ALGEBRAIC STRUCTURE

Time : 3 hours Maximum marks : 70

SECTION A — (3 × 3 = 9 marks)

Answer any THREE questions out of Five questions.

All questions carry equal marks.

1. Define an equivalence relation and give an example.
2. Show that identity element of a group is unique.
3. Show that every subgroup of an abelian group is normal.
4. Define an ideal of a ring and give an example.
5. Show that any Euclidean domain R is a principal ideal domain.

SECTION B — ($3 \times 7 = 21$ marks)

Answer any THREE questions out of Five questions.

All questions carry equal marks.

6. Show that $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x - 3$ is a bijection and find its inverse. Compute $f^{-1} \circ f$.

7. Establish that a non-empty subset H of a group G is a subgroup of G if and only if $a, b \in H \Rightarrow ab^{-1} \in H$.

8. Let $f : G \to G$ be a homomorphism. Show that the kernal K of f is a normal subgroup of G.

9. Show that a finite commutative ring R without zero-divisors is a field.

10. Let R be an integral domain. Let a and b be two non-zero elements of R. Show that a and b are associates if and only if $a = bu$ where u is a unit in R.

SECTION C — ($4 \times 10 = 40$ marks)

Answer any FOUR questions out of Seven questions.

All questions carry equal marks.

11. (a) In $\mathbb{Z}$, we define $a \rho b$ if and only if ab is a perfect square. Is ρ an equivalence relation?

    (b) Prove by induction that $2^n > n$ for all $n \in \mathbb{N}$.

12. Let A and B be two subgroups of a group G. Show that AB is a subgroup of G if and only if $AB = BA$.

13. State and prove the Lagrange's theorem.

14. (a) Show that any field is an integral domain.

    (b) Show that the characteristic of an integral domain D is either 0 or a prime number.

15. Show that any Euclidean domain R is a unique factorization domain.

16. Establish that any finite group is isomorphic to a group of permutations.

17. Show that any integral domain D can be embedded in a field F and every element of F can be expressed as a quotient of two elements of two elements of D.

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UG-CS-1351 **BMSSA–42**

U.G. DEGREE EXAMINATION – FEBRUARY 2025.

Computer Science/Computer Application

Fourth Semester

PROGRAMMING IN C

Time : 3 hours Maximum marks : 70

PART A — (3 × 3 = 9 marks)

Answer any THREE questions out of Five questions in 100 words.

All questions carry equal marks.

1. What is a Data types?

2. List out the Loop statements.

3. What is a Register variable?

4. Define : Union

5. What is a pointer?

PART B — (3 × 7 = 21 marks)

Answer any THREE questions out of Five questions in 200 words.

All questions carry equal marks.

6. Describe the Library function give example.

7. Explain the do- while statement with an example.

8. Explain the general format for defines a function.

9. Discuss about the User defined data types.

10. Write a note on pointer to function.

PART C — (4 × 10 = 40 marks)

Answer any FOUR questions out of Seven questions in 500 words.

All questions carry equal marks.

11. Describe the various types of Expressions in C.

12. Explain the following

    (a) Switch case

    (b) Do-While.

13. Describe the various data I/Ofunctions in C.

14. Write a program to sort the given set of n names in alphabetical order.

15. Discuss about the passing pointers to function.

16. Explain in detail the Conditional operators.

17. Write a C function to find the largest element in an array using pointers.

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UG-CS-1352 **BMSS–51**

U.G. DEGREE EXAMINATION – FEBRUARY 2025.

Mathematics

Fifth Semester

REAL ANALYSIS – I

Time : 3 hours Maximum marks : 70

SECTION A — (3 × 3 = 9 marks)

Answer any THREE questions out of Five questions in 100 words.

All questions carry equal marks.

1. Show that the set of all rational numbers in [0,1] is countable.

2. Define a convergent sequence and give an example.

3. Find limit inferior and limit superior of the sequence 1, 2, 1, 3, 1, 4,....

4. If $\sum_{n=1}^{\infty} a_n$ is a convergent series, then show that $\lim_{n\to\infty} a_n = 0$.

5. Is the function $f(x) = \frac{\sin x}{x}, (x \in \mathbb{R}, x \neq 0)$ continuous? Justify.

SECTION B — (3 × 7 = 21 marks)

Answer any THREE questions out of Five questions in 200 words.

All questions carry equal marks.

6. If $A \to B$ and if $X \subset B, Y \subset B$, then show that $f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$.

7. Show that every convergent sequence is bounded.

8. If $\{s_n\}_{n=1}^{\infty}$ is a Cauchy sequence of real numbers, then show that $\{s_n\}_{n=1}^{\infty}$ is convergent

9. State and prove the Cauchy condensation test.

10. Show that $\mathbb{R}^n$ is a metric space.

SECTION C — (4 × 10 = 40 marks)

Answer any FOUR questions out of Seven questions in 500 words.

All questions carry equal marks.

11. Establish that the countable union of countable set is countable.

12. Show that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$ is convergent.

13. Show that any bounded sequence of real numbers has a convergent subsequence.

14. If $s=\{s_n\}_{n=1}^{\infty}$ and $t=\{t_n\}_{n=1}^{\infty}$ are in ℓ^2, then prove that $s+t=\{s_n+t_n\}_{n=1}^{\infty}$ is in ℓ^2 and also prove that $\left[\sum_{n=1}^{\infty}(s_n+t_n)^2\right]^{1/2} \leq \left[\sum_{n=1}^{\infty}s_n^2\right]^{\frac{1}{2}}+\left[\sum_{n=1}^{\infty}t_n^2\right]^{1/2}$.

15. Show that continuous function of a continuous function is continuous.

16. If $\lim_{n\to\infty} s_n = L$, and $\lim_{n\to\infty} s_n = M$, then prove that $L=M$.

17. State and prove the root test.

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UG-CS-1353 BMSS-52

U.G. DEGREE EXAMINATION – FEBRUARY 2025.

Mathematics

Fifth Semester

LINEAR ALGEBRA

Time : 3 hours Maximum marks : 70

PART A — (3 × 3 = 9 marks)

Answer any THREE questions out of Five questions in 100 words.

All questions carry equal marks.

1. If V is a vector space over a field F, then show that

   (a) $\alpha 0 = 0$ for all $\alpha \in F$

   (b) $0v = 0$ for all $v \in V$.

2. Define a dual space of a vector space. What is the dimension of the dual space?

3. Show that $\|\alpha v\| = |\alpha|\|v\|$, $\alpha \in F, v \in V$.

4. Show that the element $\lambda \in F$ is a characteristic root of $T \in A(V)$ if and only if for some $v \neq 0, vT = \lambda v$.

5. When do you say that two linear transformations are similar?

PART B — (3 × 7 = 21 marks)

Answer any THREE questions out of Five questions in 200 words.

All questions carry equal marks.

6. Let S be a non-empty subset of a vector space V. Show that $L(S)$, the linear span of S, is a subspace of V.

7. Show that $A(W)$, the annihilator of W, is a subspace of the dual space of V.

8. If V is a finite-dimensional inner product space and W is a subspace of V, then prove that $\left(W^{\perp}\right)^{\perp} = W$.

9. If $\lambda \in F$ is a characteristic root of $T \in A(V)$, then prove that for any polynomial $q(x) \in F[x]$, $q(\lambda)$ is a characteristic root of $q(T)$.

10. Let $V = F^3$ and suppose that $\begin{pmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$ is the matrix of $T \in A(V)$ in the basis $v_1 = (1, 0, 0)$, $v_2 = (0, 1, 0)$, $v_3 = (0, 0, 1)$. Find the matrix of T in the basis $u_1 = (1, 1, 1)$, $u_2 = (0, 1, 1)$, $u_3 = (0, 0, 1)$.

PART C — (4 × 10 = 40 marks)

Answer any FOUR questions out of Seven questions in 500 words.

All questions carry equal marks.

11. If V is finite dimensional and if W is a subspace of V, then show that

(a) W is finite dimensional

(b) $\dim W \leq \dim V$

(c) $\dim\left(\frac{V}{W}\right) = \dim V - \dim W$.

12. Let V and W are vector spaces over a field F. Show that $Hom(V, W)$ is a vector space over F.

13. Let V be a finite-dimensional inner product space, then show that V has an orthonormal set as a basis.

14. If V is finite-dimensional over F, then prove that $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not 0.

15. If $T \in A(V)$ has all its characteristic root in F, then show that there is a basis of V in which the matrix of T is triangular.

16. State and prove the Schwarz inequality.

17. If A is an algebra, with unit element, over F, then show that A is isomorphic to a subalgebra of $A(V)$ for some vector space V over F.

UG-CS-1354 **BMSS–53**

U.G. DEGREE EXAMINATION – FEBRUARY 2025.

Mathematics

Fifth Semester

DISCRETE MATHEMATICS

Time : 3 hours Maximum marks : 70

PART A — ($3 \times 3 = 9$ marks)

Answer any THREE questions out of Five questions in 100 words.

All questions carry equal marks.

1. Find the gcd of 315 and 4235.

2. Define Boolean algebra.

3. Define equivalent switching circuits.

4. Define recurrence relation.

5. Define Euler circuit with and example.

PART B — (3 × 7 = 21 marks)

Answer any THREE questions out of Five questions in 200 words.

All questions carry equal marks.

6. Prove that

$$1.2 + 2.3 + 3.4 + \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}.$$

7. Show that the Boolean expressions $(x_1.x_2).x_3$ and $x_1.(x_2.x_3)$ are equal.

8. Draw the switching circuit for the Boolean expression $yz + xy'zu + zu' + xz'$.

9. The Fibonacci sequence $\{f_n\}$ is defined by $f_n = f_{n-1} + f_{n-2}, \quad n \geq 3$ with initial conditions $f_1 = f_2 = 1$. Find an explicit formula for $\{f_n\}$.

10. Prove that the sum of the degrees of all vertices of a graph is an even integer.

PART C — (4 × 10 = 40 marks)

Answer any FOUR questions out of Seven questions in 500 words.

All questions carry equal marks.

11. State and prove De Morgan's laws in set theory.

12. Express the Boolean expression $(x + y + z)(xy + x'z)'$ in DNF in the variables x, y, z.

13. Draw a circuit using only NAND-gates that the represents the Boolean functions

    (a) $f(x, y) = x + y$

    (b) $f(x, y) = xy$

14. Using generating functions solve the recurrence relation $a_n = 3a_{n-1} + 2$ for all $n \geq 1$, $a_0 = 2$.

15. If G is a connected graph and every vertex has even degree then G has an Euler circuit.

16. Prove that the following properties are equivalent in a Boolean algebra B :

(a) $a.c = a$

(b) $a.c' = 0$

(c) $a + c = c$ for all $a, c \in B$.

17. We wish a light in a room to be controlled independently by three wall switches located at the three entrances of the room in such a way that flicking any one of them will change the state of the light (on to off and off to on). Design a simple series - parallel switching circuit which will do the required job.

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UG-CS-1355 **BMSSE–51**

U.G. DEGREE EXAMINATION – FEBRUARY 2025.

Mathematics

Fifth Semester

MATHEMATICAL STATISTICS

Time : 3 hours Maximum marks : 70

PART A — ($3 \times 3 = 9$ marks)

Answer any THREE questions out of Five questions in 100 words.

All questions carry equal marks.

1. Find Geometric mean from the following data :

X	5	15	25	35	45
Y	5	7	15	25	8

2. The following are the two regression lines $8x - 10y + 66 = 0$ and $40x - 18y = 214$. Find the mean values.

3. Define :

   (a) Random variable

   (b) Moment generating function.

4. Define :

   (a) one-tailed test

   (b) two-tailed test

   (c) standard error.

5. Define student's t statistic.

PART B — (3 × 7 = 21 marks)

Answer any THREE questions out of Five questions in 200 words.

All questions carry equal marks.

6. Find mean deviation from median and its coefficient from the following data :

X	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Y	10	15	15	23	22	19	12	4

7. Find the rank correlation from the following data :

X	97	51	68	75	42	33
Y	15	53	47	28	64	82

8. If X is a random variable and a and b are constants then prove that $E(aX+b)=aE(X)+b$ provided all the expectations exist.

9. A sample of 900 members has a mean 3.4 cms and standard deviation is 2.6 cms. Is the sample from a large population of mean 3.25 cms and standard deviation 2.61 cms?

10. Two random samples gave the following results :

Sample	Size	Sample mean	Sum of squares of deviations from the mean
1	10	15	90
2	12	14	108

Test whether the samples come from the same normal population at 5% level of significance.

PART C — (4 × 10 = 40 marks)

Answer any FOUR questions out of Seven questions in 500 words.

All questions carry equal marks.

11. Find quartile deviation and its coefficient from the following data :

X	0-10	10-20	20-30	30-40	40-50	50-60
Y	8	20	25	30	12	5

12. Obtain both regression lines from the following data :

X	10	12	13	17	18	20	24	30
Y	5	6	7	9	13	15	20	21

13. State and prove Baye's theorem.

14. Random samples of 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test the hypothesis that proportions of men and women in favour of the proposal, are same against that they are not, at 5% level.

15. State and prove additive property of chi-square distribution.

16. The following tables gives the number of aircraft accidents that occurs during the various days of the week. Find whether the accidents are uniformly distribute over the week.

Days	Sunday	Monday	Tuesday	Wednesday
No. of accidents	14	16	8	12
Days	Thursday	Friday	Saturday	
No. of accidents	11	9	14	

17. The heights of six randomly chosen sailors are in inches : 63, 65, 68, 69, 71 and 72. Those of 10 randomly chosen soldiers are 61, 62, 65, 66, 69, 69, 70, 71, 72 and 73. Discuss the light that these data throw on the suggestion that sailors are on the average taller than soldiers.

UG-CS-1357 **BMSS–61**

U.G. DEGREE EXAMINATION – FEBRUARY 2025

Mathematics

Sixth Semester

REAL ANALYSIS II

Time : 3 hours Maximum marks : 70

SECTION A — (3 × 3 = 9 marks)

Answer any THREE questions out of Five questions in 100 words each.

All questions carry equal marks.

1. Prove that "every subset of R_d is open"

2. Let A be a subset of the metric space (M, ρ). If (A, ρ) is compact then prove that A is a closed subset of (M, ρ).

3. Show that every countable subset of R' has measure zero.

4. State the law of the Mean.

5. State the Second Mean-value theorem for integrals.

SECTION B — (3 × 7 = 21 marks)

Answer any THREE questions out of Five questions

All questions carry equal marks.

6. If A_1 and A_2 are connected subsets of a metric space M and if $A_1 \cap A_2 \neq \phi$ then prove that $A_1 \cup A_2$ also connected.

7. If the metric space M has the Heine-Borel property, then show that M is compact.

8. Let f be a bounded function on $[a; b]$. Then show that every upper sum for f is greater than or equal to every lower sum for f. That is if σ and τ are any two sub division of $[a; b]$ then $U[f;\sigma] \geq L[f;\tau]$.

9. State and prove Rolle's theorem.

10. State and prove Taylor's formula with integral form of the remainder.

SECTION C — (4 × 10 = 40 marks)

Answer any FOUR questions out of Seven questions in 500 words.
All questions carry equal marks.

11. Let $f : R^1 \to R^1$ and $a \in R$: Then prove that f is continuous at "a" if and only if $\omega\ [f; a] = 0$.

12. Let (M, ρ) be a metric space. Show that the subset A of M is totally bounded if and only if every subsequence of points of A contains a Cauchy subsequence.

13. Prove that if $f \in R[a;b]$ and $a < c < b$; then $f \in R[a;c]; f \in R[c;b]$ and $\int_a^b f = \int_a^c f + \int_c^b f$.

14. Suppose f has a derivative at c and that g has a derivative at $f(c)$: Then prove that $\phi = g \circ f$ has a derivative at c and $\phi'(c) = g'(f(c))\, f'(c)$.

15. State and prove Taylor's formula with Lagrange form of the remainder.

16. If the real-valued function f is continuous on the compact metric space M; then show that f attains a maximum value at some point of M: Also, f attains a minimum value at some point of M.

17. Prove that if f and g both have derivatives at $c \in R^1$ then so do $f+g; f-g; f\,g$ and we have

$(f+g)'(c) = f'(c) + g'(c)$

$(f-g)'(c) = f'(c) - g'(c)$

$(fg)'(c) = f'(c)g(c) + g(c)f'(c)$

Further, if $g'(c) \neq 0$ then f/g has a derivative at c and $\left(\frac{f}{g}\right)'(c) = \frac{g(c)f'(c) - f(c)g'(c)}{[g(c)]^2}$.

UG-CS-1358 **BMSS–62**

U.G. DEGREE EXAMINATION – FEBRUARY 2025.

Mathematics

Sixth Semester

MECHANICS

Time : 3 hours Maximum marks : 70

PART A — (3 × 3 = 9 marks)

Answer any THREE questions.

1. Define Cone of Friction.

2. Define Virtual displacement, Centre of gravity.

3. Define Conservative forces.

4. Define Horizontal range. Range of an inclined plane.

5. When the Slipping of the cycle takes place.

PART B — (3 × 7 = 21 marks)

Answer any THREE questions.

6. The sides *BC*, *CA* and *AB* of a ΔABC are bisected in *D*, *E* and *F*. Show that the forces represented by *DA*, *EB*, *FC* are in equilibrium.

7. Find the mass centre of Solid right circular cone of height h using integration.

8. The displacement *x* of a particle moving along a straight line is given by $x = A\cos nt + B\sin nt$, where *A*, *B*, *n* are constants. Show that its motion is S.H. If *A* = 3, *B* = 4, *n* = 2, find its period, amplitude, maximum velocity and maximum acceleration.

9. Show that the path of a projectile is a parabola.

10. A particle *P* of mass *m*, tied to the end of a light inelastic string of length *l*, is made describe a circle with a constant angular speed ω on a smooth horizontal table, the other end of the string being fixed at a point *O* in the table. Show that the tension in the string is $m\omega^2$.

PART C — ($4 \times 10 = 40$ marks)

Answer any FOUR questions.

11. State and prove Lami's theorem.

12. Find the mass centre using integration

    (a) Thin wire in the form of a circular arc.

    (b) Solid hemi-sphere of radius a.

13. A light spiral spring of length l hangs vertically in the position OA where O is the point of suspension. Let OB be its equilibrium position when a mass m is attached to its lower end and $AB = a$. If m is pulled vertically downward from B to C, through a distance b, find its motion.

14. Find the envelope of a family of trajectories of particles projected from a fixed point O and constant velocity u.

15. Discuss the motion of a conical pendulum.

16. Suppose a particle of mass m is placed on a rough inclined plane inclined at an angle α to the horizontal and a force of magnitude S acts on it in a direction making an angle θ with the plane. The equilibrium is limiting to find S.

17. (a) Find the velocities of two smooth spheres after a direct impact between them.

(b) Show that, when two spheres of equal masses m collide directly, the velocities of the spheres are interchanged if $e = 1$.

UG-CS-1359 **BMSS-63**

U.G. DEGREE EXAMINATION – FEBRUARY 2025.

Mathematics

Sixth Semester

COMPLEX ANALYSIS

Time : 3 hours Maximum marks : 70

SECTION A — (3 × 3 = 9 marks)

Answer any THREE questions.

1. Prove that $f(z)=z^3$ analytic and also find its derivative.

2. Define conformal mapping.

3. Let C be positively oriented circle $|z|=2$. Evaluate $\int_C \frac{zdz}{(9-z^2)(z+i)}$.

4. Write the Maclaurin's series

5. State the Rouche's theorem.

SECTION B — (3 × 7 = 21 marks)

Answer any THREE questions.

6. Show that the function $f(z) = xy + iy$ is everywhere continuous but is not analytic.

7. Find the bilinear transformation which carries $0, i, -i$ and $1, -1, 0$.

8. State and prove Liouville's theorem.

9. Expand $f(z) = \cos z$ in a Taylor's series at $z = \frac{\pi}{4}$.

10. Determine the number of roots of the equation, $z^7 - 4z^3 + z = 1$ inside the circle $|z| = 1$.

SECTION C — (4 × 10 = 40 marks)

Answer any FOUR questions.

11. Derive Cauchy Riemann Equations in polar form.

12. Prove that the transformation $w = \sin z$ maps the families of lines $x =$ constant and $y =$ constant into two families of confocal conics.

13. State and prove Cauchy integral formula.

14. If a function $f(z)$ is analytic inside and on a circle with center z_0 then prove that for any point lying with in the circle $f(z)=\sum_{n=0}^{\infty} a_n (z-z_n)^n$ where $a_n = \frac{f^n(z_0)}{n!}$.

15. State and prove Argument principle.

16. State and prove Schwarz Lemma.

17. Expand $f(z)=\frac{1}{(z-i)(z-2)}$ in a Laurent's series valid for the region

    (a) $|z|<1$

    (b) $1<|z|<2$

    (c) $|z|>2$.

UG-CS-1360 **BMSSE–61**

U.G. DEGREE EXAMINATION – FEBRUARY 2025.

Mathematics

Sixth Semester

OPERATIONS RESEARCH

Time : 3 hours Maximum marks : 70

SECTION A — (3 × 3 = 9 marks)

Answer any THREE questions out of Five questions in 100 words.

All questions carry equal marks.

1. Define optimal solution.

2. What is meant by assignment problem?

3. Define saddle point.

4. Write a note on queueing system

5. Write the three time estimates in PERT.

SECTION B — (3 × 7 = 21 marks)

Answer any THREE questions out of Five questions in 200 words.

All questions carry equal marks.

6. Solve the following LPP by graphical method.

Maximize $Z = 5x_1 + 7x_2$

Subject to the constraints $x_1 + x_2 \leq 4$, $3x_1 + 8x_2 \leq 24$, $10x_1 + 7x_2 \leq 35$ and $x_1, x_2 \geq 0$.

7. Write the procedure for Hungarian Assignment method.

8. Solving the following 2X2 two person zero-sum game without saddle point by algebraic method

(Player B)

(Player A) $\begin{pmatrix} 0 & -3 \\ -0 & 0 \end{pmatrix}$

9. Briefly discuss the application of queueing theory in various fields.

10. Explain general procedure for the construction of net work diagram.

SECTION C — (4 × 10 = 40 marks)

Answer any FOUR questions out of Seven questions in 500 words.

All questions carry equal marks.

11. Solve the following LPP by simplex method.

Maximize $Z = 3x_1 - x_2$

Subject to the constraints $2x_1 + x_2 \geq 2$, $x_1 + 3x_2 \leq 3$, $x_2 \leq 4$ and $x_1, x_2 \geq 0$.

12. Solve the transportation problem by using least cost method.

	A	B	C	D	E	Supply
P	4	1	2	6	9	100
Q	6	4	3	5	7	120
R	5	2	6	4	8	120
Demand	40	50	70	90	90	

13. Solve the following 2×4 game by graphical method

$$\begin{array}{c} \\ \text{(Player A)} \end{array} \begin{array}{c} \text{(Player B)} \\ \begin{pmatrix} 1 & 0 & 4 & -1 \\ -1 & 1 & -2 & 5 \end{pmatrix} \end{array}$$

14. A supermarket has two girls ringing up sales at the counters. If the service time for each customer is exponential with mean 4 minutes, and if the people arrive in a Poisson fashion at the rate of 10 per hour.

    (a) What is the probability of having to wait for services?

    (b) What is the excepted percentage of idle time for each girl?

    (c) If a customer has to wait, what is the excepted length of his waiting time?

15. A project schedule has the following characteristics. Activity Expected duration (weeks)

	Expected duration (Weeks)		
Activity	Optimistic	Most likely	Pessimistic
1-2	3	3	3
2-3	3	6	9

2-4	2	4	6
3-5	4	6	8
4-6	4	6	8
5-6	0	0	0
5-7	3	4	5
6-7	2	5	8

(a) Draw the project network and trace all the possible paths from it.

(b) Determine the critical path and the expected project time.

(c) What is the probability that will be completed in 20 weeks?

16. Solve the assignment problem for assigning five machines to five jobs.

		Machines				
		1	2	3	4	5
Jobs	A	9	22	58	11	19
	B	43	78	72	50	63
	C	41	28	91	37	45
	D	74	42	27	49	39
	E	36	11	57	22	25

17. Find the sequence that minimizes the total elapsed time required to complete the following tasks.

Also find the idle time on each machine

Jobs	A	B	C	D	E	F	G
Machine M1	3	12	15	6	10	11	9
Machine M2	8	10	10	6	12	1	3